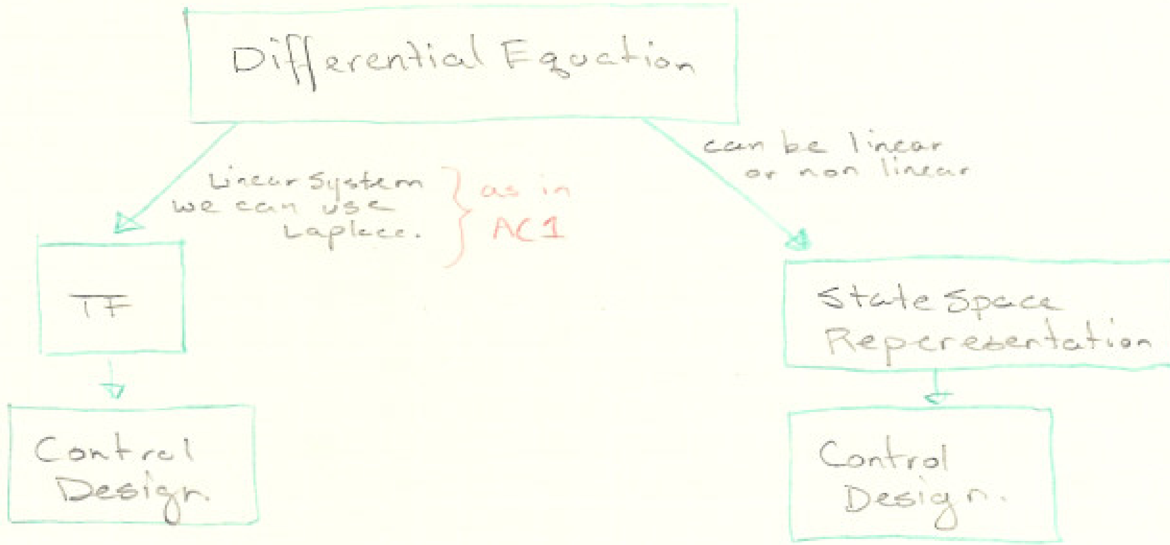


STATE SPACE REPERSENTATION:

$$\ddot{y} + 2\dot{y} + 3y = u$$

introduce new variables.

$$x_1 = y$$

$$x_2 = \dot{y}$$

$$\therefore \quad \dot{x}_1 = \dot{y} = x_2 \quad , \quad \dot{x}_2 = \ddot{y} = -3y - 2\dot{y} + u = -3x_1 - 2x_2$$

$$\ddot{y} = -3y - 2\dot{y} + u = -3x_1 - 2x_2 + u$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -3x_1 - 2x_2 + u$$

x_1, x_2 are state variables.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\dot{x} = A \quad x + B$$

$$\boxed{\dot{x} = Ax + Bu}$$

note: x is known as the state vector.

$$\boxed{y = Cx + Du}$$

note: in most practical systems, D matrix is zero.

note: the eigen values of the A matrix are the poles of the original system.

Ex: calculating eigen values of A.

λ_1, λ_2 are the eigen values of A.

$$\boxed{\det(\lambda I - A) = 0}$$

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ 3 & \lambda + 2 \end{bmatrix}$$

$$= \lambda(\lambda + 2) + 3 = 0$$

$$\boxed{= \lambda^2 + 2\lambda + 3 = 0}$$ same as char equ

for continuous time LTI systems, we can write

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$X \in \mathbb{R}^n \Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$u \in \mathbb{R}^m \Rightarrow$ control input.

$Y \in \mathbb{R}^p \Rightarrow$ output vector.

$A \in \mathbb{R}^p \Rightarrow$ output vector.

$A \in \mathbb{R}^{n \times n} \Rightarrow$ state matrix

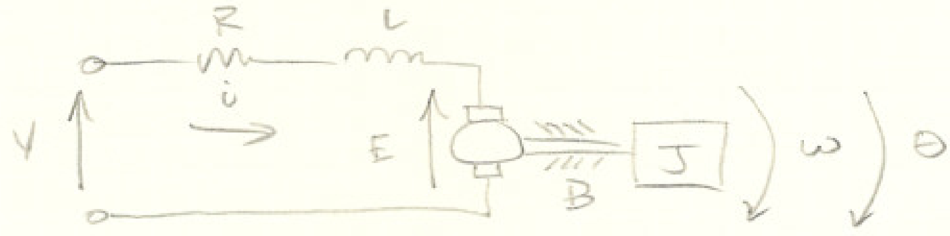
$B \in \mathbb{R}^{n \times m} \Rightarrow$ input matrix

$C \in \mathbb{R}^{p \times n} \Rightarrow$ output vector.

$D \in \mathbb{R}^{p \times m} \Rightarrow$ direct transmission matrix.

note: the only time "D" is different from zero is when the numerator of the transferfunction has an order equal or greater then the denomanator.

DC MOTOR



$$V = Ri + L\frac{di}{dt} + E \qquad (1)$$

$$T_m - T_L - B\omega = J\ddot{\theta} \qquad (2)$$

$$E = k_1 \omega \qquad (3)$$

$$T_m = k_2 i \qquad (4)$$

$$\omega = \frac{d\theta}{dt} \qquad (5)$$

$$(1) \Rightarrow \quad \frac{di}{dt} = -\frac{R}{L} i - \frac{k_1}{L}\omega + \frac{V}{L}$$

$$(2) \Rightarrow \quad \frac{d\omega}{dt} = -\frac{B}{J}\omega + \frac{k_2}{J} i - \frac{T_L}{J}$$

$$(3) \Rightarrow \quad \frac{d\theta}{dt} = \omega$$

$$x = \left.\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix}\right\} \text{state vector.}$$

T_L: disturbance, ∴ let it be zero.

$u = V$

$$\dot{x} = Ax + Bu$$

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{k_1}{L} & 0 \\ \frac{k_2}{J} & -\frac{B}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix}$$

for position control.

$$y = \theta = Cx$$

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

for control of θ, ω.

$$y = \begin{bmatrix} \omega \\ \theta \end{bmatrix} = Cx$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$